

# AP Calculus Summer Work

I am so excited to have you in AP Calculus this year! Calculus brings all the math you've learned throughout the years together and allows you to evaluate and justify real world situations mathematically. In the following pages you will find a few review topics we want to discuss in the first few days of the year. We will briefly review the topics you struggled with and will have some time to revise your work before going over the solutions and making corrections. Your goal is to complete each section to the best of your ability, reviewing topics that will be imperative to your success next year. Keep in mind, this should not take over your life this summer. Take the much needed break, have some fun and recharge after last school year. Ideally, you are working on this as the summer is wrapping up and you are preparing to get back in the school mindset.

At the beginning of each section are links for helpful videos to help you with corresponding sections in the summer work packet. Use these videos as references to recall material covered in PreCalculus. Some topics may be from courses prior to PreCalculus, so they may be more challenging to recall. Do your best to complete, or at least make a solid attempt in all of the sections. Come ready with questions and we will fill in the gaps together!

Remember, the goal is not to come in with this 100% ready to turn in, but with a solid attempt for each section. If you have any questions, please feel free to email me over the summer break and I will get back to you as quickly as possible!

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## Linear Review

### Video resources

Horizontal and vertical lines - <https://www.youtube.com/watch?v=2GYbX7PbmzA>

Parallel and perpendicular line - <https://www.youtube.com/watch?v=LTb2-LE7StE>

Line tangent to a circle - [https://www.youtube.com/watch?v=ucEghF7\\_HvU](https://www.youtube.com/watch?v=ucEghF7_HvU)

Show all work. No Calculator

1. Write an equation of the (a) vertical line and (b) horizontal line passing through the point  $(-7, \pi)$ .
2. Write an equation of the line in **slope-intercept form** passing through the points  $(-3, 4)$  and  $(5, -1)$ .
3. Write an equation of the line through the point  $(4, \frac{1}{2})$  that is
  - (a) parallel to  $4x - 3y = 6$
  - (b) perpendicular to the line  $4x - 3y = 6$
  - (c) find both the  $x$ - and  $y$ - intercepts of the line found in part (b).

4. Find the value of  $x$  for which line through  $(-8, -3)$  and  $(x, 4)$  has a slope of 3. (**Recall/hint:**  $m = \frac{y_2 - y_1}{x_2 - x_1}$ )

5. Write the equation of the line that is tangent to the circle at the given point. (**Recall/hint:** standard form for a circle  $(x - h)^2 + (y - k)^2 = r^2$ , where  $(h, k)$  is the center point of the circle and the radius and tangent lines are perpendicular at the point of tangency)

$$(x - 2)^2 + (y - 1)^2 = 25; (6, -2)$$

6. Which of the following is an equation of the line through  $(-3, 4)$  with a slope of  $\frac{1}{2}$ ? (show the work/thought process that leads you to your answer choice)

$$\begin{array}{lll} \text{(A) } y - 4 = \frac{1}{2}(x + 3) & \text{(B) } y + 3 = \frac{1}{2}(x - 4) & \text{(C) } y - 4 = -2(x + 3) \\ \text{(D) } y - 4 = 2(x + 3) & \text{(E) } y + 3 = 2(x - 4) & \end{array}$$

7. Which of the following is an equation of the vertical line through  $(2, -4)$ ? (show the work/thought process that leads you to your answer choice)

$$\text{(A) } y = -4 \quad \text{(B) } x = -2 \quad \text{(C) } y = 4 \quad \text{(D) } x = 0 \quad \text{(E) } x = 2$$

## Rational and Radical Functions

### Video resources

Simplifying rational expressions - <https://www.youtube.com/watch?v=7Uos1ED3KHI>

Adding and subtracting rational expressions - [https://www.youtube.com/watch?v=y\\_DweTAEYWk](https://www.youtube.com/watch?v=y_DweTAEYWk)

Multiplying and dividing rational expressions - [https://www.youtube.com/watch?v=x\\_5hDLe8UL0](https://www.youtube.com/watch?v=x_5hDLe8UL0)

Solving rational expressions - [https://www.youtube.com/watch?v=1fR\\_9ke5-n8](https://www.youtube.com/watch?v=1fR_9ke5-n8)

<https://www.youtube.com/watch?v=tynJHA7wFzA>

Solving radical equations - <https://www.youtube.com/watch?v=g3rzugglgIw>

### Simplify and determine where the simplified function is undefined

$$1. \frac{4x+16}{2x+6} \times \frac{x^2+2x-3}{x+4}$$

$$2. \frac{x^2-2x-8}{x^2-2x-15} \div \frac{2x^2-8x}{2x^2-10x}$$

$$3. \frac{\frac{x+3}{x-1}}{\frac{x^2+5x+6}{x^2-2x+1}}$$

$$4. \frac{x+4}{x^2-x-12} + \frac{2x}{x-4}$$

$$5. \frac{3x^2-1}{x^2-3x-18} - \frac{x+2}{x-6}$$

**Solve. Check your solution.**

6.  $\frac{3-x}{x^2-9} = -3$

7.  $\frac{4x}{x^2-4} = \frac{1}{x-2}$

**Solve each equation.**

8.  $\sqrt{x+4} = 3\sqrt{x}$

9.  $\sqrt[3]{x-6} = \sqrt[3]{3x+24}$

10.  $\sqrt{-14x+2} = x-3$

## Exponential and Logarithmic Functions

### Video resources

Log properties – <https://www.youtube.com/watch?v=mQTWzLpCcW0>  
<https://www.youtube.com/watch?v=Jtv9Lnf7Zw8>

Expanding logarithms - <https://www.youtube.com/watch?v=Olz-5MyJA3g>

Condensing logarithms - <https://www.youtube.com/watch?v=luRrOlsB4cY>

Solving log equations – <https://www.youtube.com/watch?v=Kv2iHde7Xgw>  
<https://www.youtube.com/watch?v=eTWCARmrzJ0>

Solving exponential equations - <https://www.youtube.com/watch?v=9tutJ5xrRwg>

### Expand each logarithm

1.  $\log \frac{\sqrt{xy^3}}{z^2}$

2.  $\log \left( \frac{\sqrt[3]{xy^6}}{(zk)^2} \right)$

### Condense each logarithmic expression and simplify when possible

3.  $\log u - \log x - 3 \log w + 2 \log y$

4.  $\left( \frac{1}{2} \log x + \frac{1}{2} \log y \right) - (\log 2 - 4 \log z)$

**Solve the following exponential and logarithmic equations**

5.  $12^{2x-8} = 15$

6.  $25^x = 125^{x-2}$

7.  $\left(\frac{1}{32}\right)^{2x} = 64$

8.  $\log(t+3) + \log(t) = 1$

9.  $\log_2(t+1) + \log_2(t-1) = 5$

10.  $-2 = \log(2) - \log(3+x)$

**Simplify the following logarithms**

11.  $\ln e^{7x} + \ln e^{x+4} - e^{\ln x}$

12.  $e^{3 \ln x} - 2 \ln e^x$

## Trig Review

### Video resources

Unit circle – [https://www.youtube.com/watch?v=V5ArB\\_GFGYQ](https://www.youtube.com/watch?v=V5ArB_GFGYQ)  
<https://www.youtube.com/watch?v=wT4xMAssvDk>

Graphing sine and cosine – <https://www.youtube.com/watch?v=bvTR9XbL1co>  
<https://www.youtube.com/watch?v=KPvon2ufyNs>

Graphing other trig equations – [https://www.youtube.com/watch?v=fo\\_q9mEAFp4](https://www.youtube.com/watch?v=fo_q9mEAFp4)  
<https://www.youtube.com/watch?v=sKoR8KHx4uU>

Fundamental trig identities – <https://www.youtube.com/watch?v=ep5vjIY5kqE>  
[https://www.youtube.com/watch?v=UFb\\_LQxeGfg](https://www.youtube.com/watch?v=UFb_LQxeGfg)

Sum and difference identities – [https://www.youtube.com/watch?v=d0iUBZz\\_WIE](https://www.youtube.com/watch?v=d0iUBZz_WIE)  
<https://www.youtube.com/watch?v=hdSvvW0hpeE>

Double and half angle identities – <https://youtu.be/-Wu6ekO2Sdw>  
<https://youtu.be/bNoJC7R2c0I>  
<https://youtu.be/Xlel4XLomn4>

Trig equations - <https://youtu.be/8wa-PysIIDw>  
[https://youtu.be/gC\\_pqtQ9z0U](https://youtu.be/gC_pqtQ9z0U)  
<https://youtu.be/mzJZRInvevc>  
[https://youtu.be/JioF8Vm\\_M60](https://youtu.be/JioF8Vm_M60)  
<https://youtu.be/hHUSsGP02Ik>

Find the exact value of the following.

1.  $\csc(-270^\circ)$
2.  $\sec\left(\frac{11\pi}{3}\right)$
3.  $\tan(480^\circ)$
4.  $\sin\left(-\frac{10\pi}{3}\right)$

**Determine the amplitude, period, intervals, phase shift, and midline for each. Sketch the graph of each function for ONE CYCLE.**

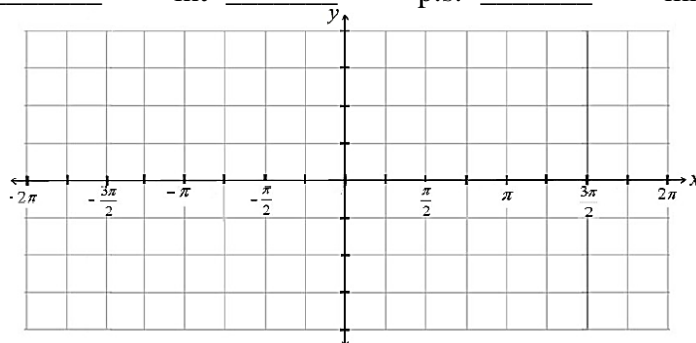
5.  $f(x) = -2\sin\left(3x - \frac{\pi}{2}\right) - 1$

amp=\_\_\_\_\_ pd=\_\_\_\_\_ int=\_\_\_\_\_ p.s.=\_\_\_\_\_ mid=\_\_\_\_\_



6.  $f(x) = 2 + \cos\frac{1}{2}(x + 2\pi)$

amp=\_\_\_\_\_ pd=\_\_\_\_\_ int=\_\_\_\_\_ p.s.=\_\_\_\_\_ mid=\_\_\_\_\_





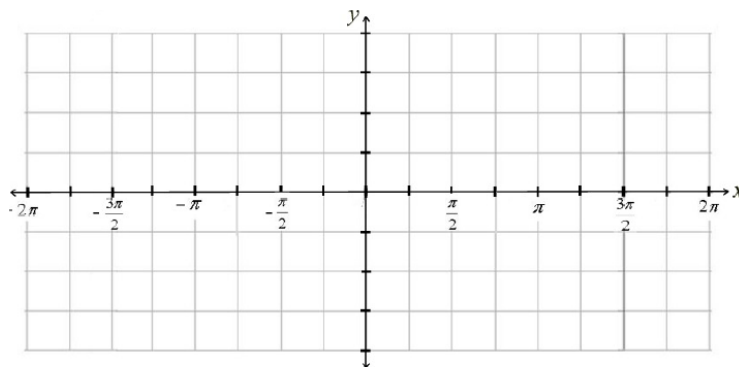
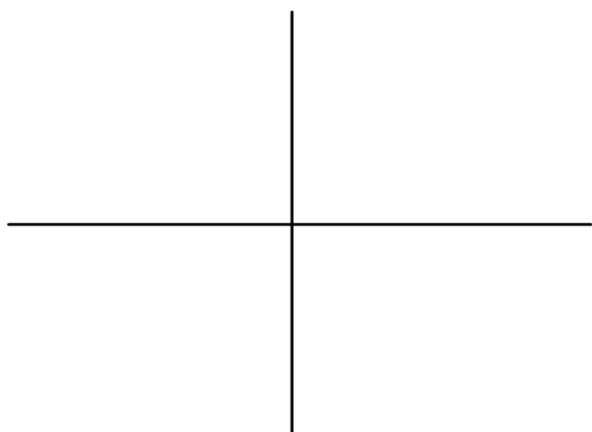
Determine the period, x-intercepts, and/or asymptotes for the following functions. Graph at least one cycle in either direction for tangent and cotangent, and one positive cycle for secant and cosecant.

7.  $f(x) = -\tan 3x$

8.  $f(x) = 5\cot \frac{2}{3}x$

pd = \_\_\_\_\_ x-int = \_\_\_\_\_ asm = \_\_\_\_\_

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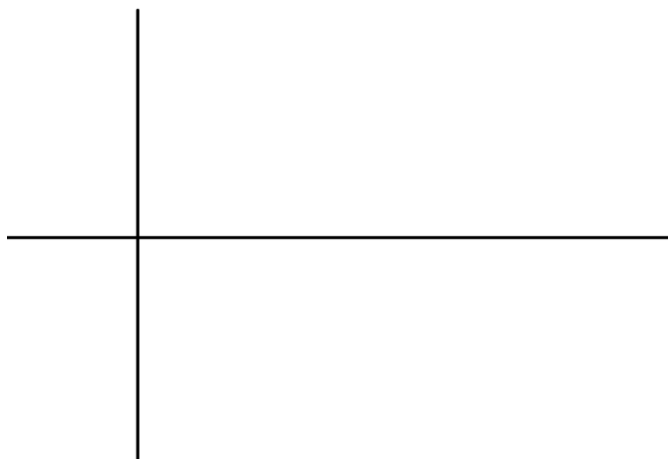
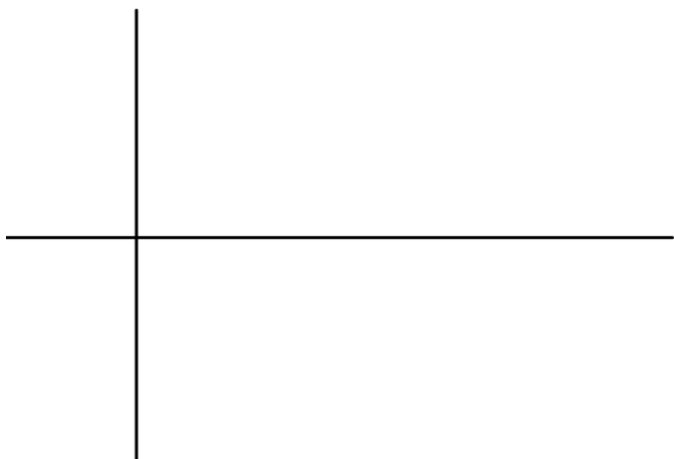


9.  $f(x) = \frac{1}{2}\csc 3x$

10.  $f(x) = \sec \frac{1}{2}x$

pd = \_\_\_\_\_ asm = \_\_\_\_\_

pd = \_\_\_\_\_ asm = \_\_\_\_\_



11. Prove  $\tan x \sin x + \cos x = \sec x$

12. Prove  $\frac{1 - \cot^2 x}{\tan^2 x - 1} = \cot^2 x$

13. Prove  $\cos\left(x - \frac{\pi}{2}\right) = \sin x$

14. Prove  $\cos 2\theta = \frac{\sin 2\theta - \tan \theta}{\tan \theta}$

**Find the exact value of each expression using a sum or difference identity.**

15.  $\sin 165^\circ$

**Find the exact value of each expression using a half angle identity.**

16.  $\cos 105^\circ$

**Find all of the solutions of each equation.**

17.  $3\sin \theta = \sin \theta - \sqrt{3}$

18.  $5\cos \theta = 3\cos \theta - 1$

**Solve each equation for the given domain.**

19.  $\cos^2 \theta - 2\cos \theta = -1$  for  $0 \leq \theta \leq 2\pi$

20.  $4\sin^2 \theta = 3\sin \theta$  for  $0^\circ \leq \theta \leq 360^\circ$

**Solve each equation for the given domain. Use trigonometric identities.**

5.  $\cos^2 \theta = \sin^2 \theta + \sin \theta$  for  $0 \leq \theta \leq 2\pi$

6.  $\cos 2\theta + \sin \theta = 1$  for  $0^\circ \leq \theta \leq 360^\circ$